ANALYSIS OF MODEL RELATIONS FOR TURBULENT HEAT DIFFUSION AS APPLIED TO THE CALCULATION OF WAKE FLOWS

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An experimental test of the known approximations is made and a new relation is proposed for turbulent diffusion of heat or of a passive admixture.

For a number of scientific—technical problems, in the investigation of the process of turbulent mixing of reagents and the construction of efficient and economical mixers, in particular, it is necessary to have detailed information about the intensity of pulsations of a passive admixture ($\overline{\theta^2}$) and about the characteristic scale τ of the scalar field. Such information could be obtained, for example, by a numerical calculation on the basis of some differential $q^2 - \varepsilon_u - \overline{\theta^2} - \varepsilon_\theta$ model. A sufficiently validated $\overline{\theta^2} - \varepsilon_\theta$ model suitable for practical calculations is not known at present, however. Along with the traditional problems connected with the nonclosure of the initial Reynolds equations, in the case of a field of a passive admixture additional difficulties arise due to its relative independence from the velocity field. In particular, as was shown in [1], no one-to-one connection exists between the characteristic temporal and spatial scales of these fields. At the same time, formal methods of constructing the closing relations define the unknown moments (e.g., $\overline{u_i\theta^2}$) to within the time scale. Here it is not clear which of the scales (dynamic, thermal, or a combination of them) must be used in a given concrete approximation. Examples of the use of various scales can be found in [2, 3]. Thus, for each of the mixed moments (e.g., $\overline{u_iu_j\theta}$) one can propose at least twice as many semiempirical approximations as for the corresponding moments of the velocity field (e.g., $\overline{u_iu_ju_k}$). Progress in the area of experimental technology now permits the direct measurement of both the model quantities and those through which they are expressed. As a result, it becomes possible to make a sound choice of the best approximation as applied to a certain class of flows.

In the present paper we confine ourselves to an analysis of model relations for the moment $\overline{u_2\theta^2}$, which describes turbulent heat diffusion in the equation for the intensity of temperature pulsations $\overline{\theta^2}$. In [4], based on the results of measuring the characteristics of an axisymmetric wake behind an ellipsoid of revolution (when the velocity of jet discharge from the rear opening corresponded to a zero excess impulse), a comparative analysis was made of the following approximations:

$$\overline{u_i\theta^2}=\alpha\tau\,\overline{u_iu_k}\,\frac{\partial\overline{\theta^2}}{\partial x_k}, \tag{1}$$

$$\overline{u_i\theta^2}=\beta\tau\left(\overline{u_iu_k}\,\frac{\partial\overline{\theta^2}}{\partial x_k}+2\overline{u_k\theta}\,\frac{\partial\overline{u_i\theta}}{\partial x_k}\right). \tag{2}$$

Dynamic ($\tau_u = q^2/\varepsilon_u$) or thermal ($\tau_\theta = \overline{\theta^2}/\varepsilon_\theta$) scales are used as τ.

It was established that the model relations (1)-(2) do not provide sufficiently close agreement with experiment. From considerations of dimensionality and tensor invariance, a model of turbulent diffusion,

$$\overline{u_i\theta^2}=\gamma\tau\left(\overline{u_iu_k}\,\frac{\partial\overline{\theta^2}}{\partial x_k}+\delta\overline{\theta^2}\,\frac{\partial\overline{u_iu_k}}{\partial x_k}\right), \tag{3}$$

which is in satisfactory agreement with test data, was proposed in [4].

Obviously, to justify one or another model relation it is not enough to compare it with the results of one experiment. In this connection, we used the results of the investigation

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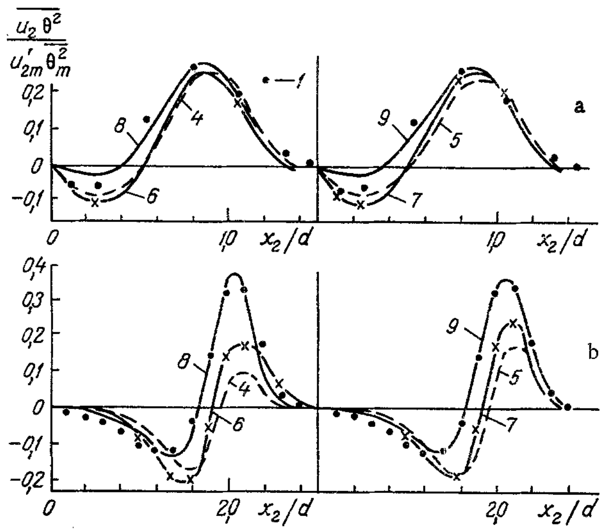

Fig. 1

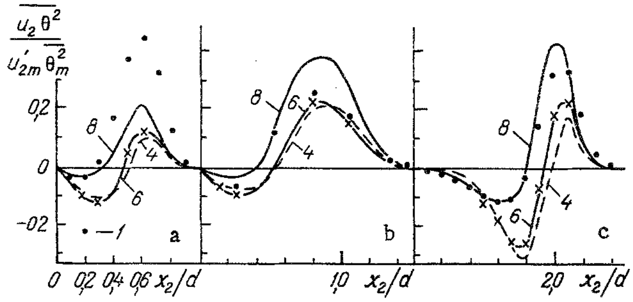

Fig. 2

Fig. 1. Experimental and calculated profiles of $\overline{u_2\theta^2}$ in an axisymmetric wake with a negative excess impulse ($x_1/d = 50$) (a) and in a plane wake ($x_1/d = 41$) (b): 1) experiment; 4-9) calculation (the numbers of the curves correspond to the numbers of the approximations).

Fig. 2. Comparison of approximations (4), (6), and (8) with experimental results (choice of constants based on variant IV): a) axisymmetric wake without impulse; b) axisymmetric wake with negative excess impulse; 3) plane wake; 1) experiment; 4, 6, 8) calculation (the numbers of the curves correspond to the numbers of the approximations).

not only of an axisymmetric wake with a zero excess impulse but also of other experiments for analysis in the present paper. One of these was carried out in the wake behind an elongated ellipsoid of revolution [5, 6], but in contrast to [4], the velocity of jet discharge was low, and the excess impulse was negative, which was responsible for basic features in the evolution of the pulsation characteristics downstream in comparison with a wake without impulse [4]. In another experiment the wake behind a plane symmetric wing profile was investigated under the conditions of temperature stratification of the medium. The measurements were made in a wind tunnel with a closed working section 82 × 82 × 1250 mm in size. A linear vertical temperature distribution was created with a grid of electrically heated elements located in the forechamber of the tunnel. A detailed description of the experimental apparatus is contained in [7]. The results of an investigation of the pulsation characteristics of a dynamic wake are given in [8]. Turbulent mixing in the wake region led to an S-shaped distortion of the temperature profile: The temperature gradient decreased near the axis of the wake while it increased at the periphery. Temperature pulsations were generated in the process. The measurements of $\overline{u_2\theta^2}$ given in the present paper were made for a temperature gradient of 0.65° C/cm in the undisturbed stream and a velocity $U_\infty = 14$ m/sec of the oncoming stream. In the investigated range of distances downstream ($x_1/d \leq 100$) the dimensionless age t_W/N of the wake (where N is the Brunt–Väisälä frequency) did not exceed 0.028, which is far lower than the critical value of 0.1 after which Archimedes forces start to affect the dynamics of the wake. The same apparatus and measurement procedure were used in all three experiments (see [9]).

Concrete approximation relations for the investigated moment $\overline{u_2\theta^2}$ are obtained from the general tensor expressions (1)-(3). In the boundary-layer approximation they have the following form:

$$\overline{u_2\theta^2}=\alpha_u\frac{\overline{u_2^2}\,q^2}{\varepsilon_u}\,\frac{\partial\overline{\theta^2}}{\partial x_2}, \tag{4}$$

$$\overline{u_2\theta^2}=\alpha_\theta\frac{q^2\overline{\theta^2}}{\varepsilon_\theta}\,\frac{\partial\overline{\theta^2}}{\partial x_2}, \tag{5}$$

$$\overline{u_2\theta^2}=\beta_u\frac{q^2}{\varepsilon_u}\left(\overline{u_2u_2}\,\frac{\partial\overline{\theta^2}}{\partial x_k}+2\overline{u_2\theta}\,\frac{\partial\overline{u_2\theta}}{\partial x_2}\right), \tag{6}$$

TABLE 1. Optimum Values of Empirical Constants in Approximations (4)-(9) and Corresponding Root-Mean-Square Deviations (%)

Variant	α_u	σ	β_u	σ	γ_u	δ_u	σ
I	—0,13	20	—0,1	19	—0,135	1,79	1,8
II	—0,105	5,2	—0,075	5,2	—0,0542	1,33	2,9
III	—0,052	12	—0,049	10	—0,0748	1,1	3
		23		21			14
IV	—0,092	5,8	—0,065	5,4	—0,073	1,41	6
		14		11			4,6

Variant	α_θ	σ	β_θ	σ	γ_θ	δ_θ	σ
I	—0,49	17	—0,34	16	—0,23	2,5	3
II	—0,32	5,4	—0,25	5,3	—0,14	1,87	3
III	—0,16	11	—0,13	8	—0,16	1,04	2,3
		21		19			13
IV	—0,24	6,1	—0,19	5,7	—0,18	1,33	3,5
		12		10			6

$$\overline{u_2\theta^2}=\beta_\theta\frac{\overline{\theta^2}}{\varepsilon_\theta}\left(\overline{u_2u_2}\frac{\partial\overline{\theta^2}}{\partial x_2}+2\overline{u_2\theta}\frac{\partial\overline{u_2\theta}}{\partial x_2}\right), \tag{7}$$

$$\overline{u_2\theta^2}=\gamma_u\frac{q^2}{\varepsilon_u}\left(\overline{u_2u_2}\frac{\partial\overline{\theta^2}}{\partial x_2}+\delta_u\overline{\theta^2}\frac{\partial\overline{u_2u_2}}{\partial x_2}\right), \tag{8}$$

$$\overline{u_2\theta^2}=\gamma_\theta\frac{q^2}{\varepsilon_\theta}\left(\overline{u_2u_2}\frac{\partial\overline{\theta^2}}{\partial x_2}+\delta_\theta\overline{\theta^2}\frac{\partial\overline{u_2u_2}}{\partial x_2}\right). \tag{9}$$

Using these relations, from the experimental values of the parameters $\overline{\theta^2}$, $\overline{u_2^2}$, $\overline{u_2\theta}$, q^2, ε_u, and ε_θ we calculated the profile of the moment $\overline{u_2\theta^2}$, which we then compared with the directly measured profile. For convenience in calculating the derivatives, the distributions of the characteristics across the wake obtained in the experiment were approximated by analytic curves. The moment $\overline{u_2\theta^2}$ was normalized to the maximum levels of the intensities of velocity and temperature pulsations, $u'_{2m}=\sqrt{(\overline{u_2^2})_{\max}}$, $\theta'_m=\sqrt{(\overline{\theta^2})_{\max}}$, in the wake cross sections under consideration. The optimum values of the constants for all the approximations were found from the condition of a minimum of the mean-square deviation of the experimental points from the calculated curves, $\sigma^2=\frac{1}{n}\sum_{k=1}^{n}(\overline{u_2\theta^2}_{\text{exp}}-\overline{u_2\theta^2}_{\text{calc}})_k^2$. In Fig. 1 we give the results of measuring the moment $\overline{u_2\theta^2}/u'_{2m}\theta_m^{\prime 2}$ in an axisymmetric and a plane wake, as well as the distributions of this quantity calculated from Eqs. (4)-(9). The optimum values of the constants in (4)-(9) for each experiment, as well as the corresponding values of the rms deviation σ (%), are given in Table 1. The number of the variant in Table 1 corresponds to the condition for choosing the constants: I) from the results of the experiment in an axisymmetric wake with a zero excess impulse; II) in an axisymmetric wake with a negative excess impulse; III) in a plane wake in the presence of temperature stratification of the medium. Variant IV corresponds to choosing the constants from the condition of a minimum of the rms deviation for all three experiments at once. For this case the values of σ for each of the experiments I-III are given in Table 1.

As we can see, for all the types of flows analyzed, the moment $u_2\theta^2$ calculated from Eqs. (8) and (9) proposed by the authors agrees far better with the experimental data than when the well-known relations (4)-(7) are used. The empirical constants α, β, γ, and δ varied from experiment to experiment, however, and they cannot be considered as universal. When the same values of the constants are chosen for all three types of flows (variant IV, Fig. 2), the ap-

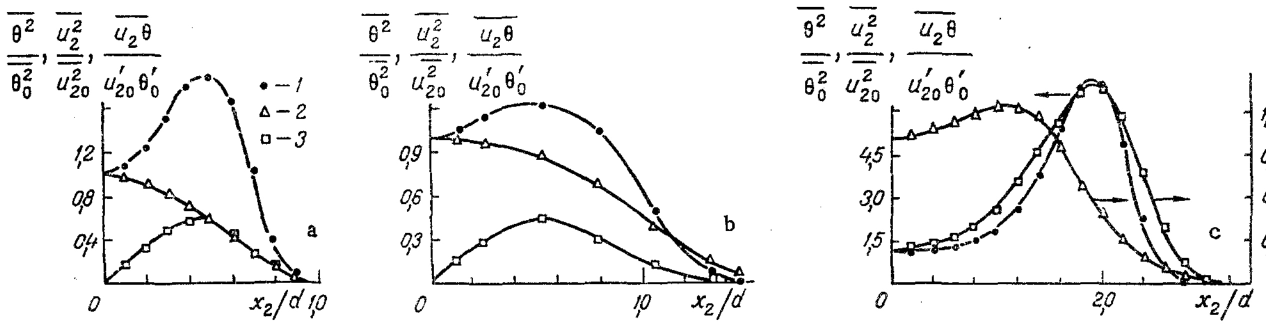

Fig. 3. Experimental profiles of $\overline{u_2^2}$, $\overline{\theta^2}$, and $\overline{u_2\theta}$: a) axisymmetric wake without impulse; b) axisymmetric wake with negative excess impulse; c) plane wake; 1) $\overline{\theta^2}/\overline{\theta^2}_0$; 2) $\overline{u_2^2}/(\overline{u_2^2})_0$; 3) $\overline{u_2\theta}/(u_2{}'_0\theta_0{}')$.

proximations (8) and (9) also prove to be the best, although the values of σ grow considerably in this case. All the same, they are lower than for the approximations (4)-(7), as can be seen from Table 1, even with the optimum choice of the constants for each experiment. Thus, regardless of whether the constants are chosen individually for each flow or for all three simultaneously, the advantage of Eqs. (8) and (9) over (4)-(7) is retained. At the same time, the results of calculations from Eqs. (8) and (9) practically coincide, i.e., these investigations do not enable us to give preference to the dynamic or the thermal scale. In the future it will be advisable to compare the proposed approximations based on the results of measurements in a number of successive cross sections of a stream in which the ratio of these scales varies downstream.

Let us analyze the reasons for the advantages of the approximations (8) and (9) over Eqs. (4)-(7). According to the available experimental data, in all the wake flows the coordinate of the maximum of the temperature pulsations usually lies at a greater distance from the axis of symmetry of the stream than the point of a change in the sign of the moment $\overline{u_2\theta^2}$. Therefore, the values of $\overline{u_2\theta^2}$ calculated from the gradient approximations (4) and (5) will differ considerably from the experimentally measured values for any assignment of the scale. Complicating the model equations with an additional term $\overline{u_2\theta}(\partial\overline{u_2\theta}/\partial x_2)$ cannot markedly improve the situation, since both terms in Eqs. (6) or (7) are distributed across the wake in the similar way (as the calculations show). In particular, the coordinates of the maxima of $\overline{\theta^2}$ and $\overline{u_2\theta}$ practically coincide, as can be seen from Fig. 3. At the same time, the maximum of the velocity pulsations $\overline{u_2^2}$ in the investigated flows lies either on the axis (Fig. 3a, b) or at a considerably smaller distance from the axis than for the profiles of $\overline{\theta^2}$ and $\overline{u_2\theta}$ (Fig. 3c). It is just the qualitative differences in the distributions of the characteristics $\overline{\theta^2}$ and $\overline{u_2^2}$ across the wake that enable us to obtain satisfactory agreement between the values of the moment $\overline{u_2\theta^2}$ calculated from Eqs. (8) and (9) and measured experimentally.

NOTATION

u_i, component of velocity pulsation; θ, temperature pulsation; $q^2 = \overline{u_iu_i}$, double kinetic energy of velocity pulsations; $\varepsilon_u = \nu\overline{(\partial u_i/\partial x_k)^2}$, rate of dissipation of kinetic energy of turbulence; $\varepsilon_\theta = \kappa\overline{(\partial\theta/\partial x_k)^2}$, rate of "dissipation" of the dispersion of temperature pulsations; x_1, x_2, longitudinal and transverse coordinates; d, maximum transverse size of the body. Indices: i = 1, 2, 3; 0, value of the quantity at the axis of the wake; ( )', root-mean-square value.

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CALCULATION OF GAS MOTION AND HEAT TRANSFER AT THE PERIPHERY OF A CYCLONE STREAM

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Methods of generalizing test data and calculating friction and convective heat exchange at the lateral surface of a cyclone chamber are discussed on the basis of an analysis of flow at the boundary of the wall region.

The investigation of convective heat exchange in the wall region of a cyclone stream is very timely at present. It has been established that the action of inertial mass forces on the hydrodynamics leads to considerable intensification of heat transfer to the lateral surface of a chamber in comparison with straight flows under similar conditions [1, 2]. In this connection, there is a considerable interest in generalizing the available test data on heat transfer from unified standpoints, allowing for the peculiarities of the hydrodynamics of a swirled stream.

We will take the swirled stream in the core of the flow out to the end surfaces as axisymmetric, incompressible, quasiisothermal, and with constant physical properties. The highest velocity w_φ in the core of cyclone flow varies along the radius r in a complicated way. We assign the characteristic form of the distribution $w_\varphi = w_\varphi(r)$ in the peripheral region, just as in the axial region [3], using a generalized approximating function,

$$\bar{w} = \left(\frac{2\eta}{1+\eta^{\varkappa}}\right)^{n}, \tag{1}$$

where n is a coefficient determined by the conditions of generation of the swirling and κ is a constant.

The distribution of the tangential velocity component at $1 \leq \eta \leq \eta_c$ can be calculated from Eq. (1), in which n is found from the condition of a maximum of $\bar{\Gamma} = \bar{w}\eta$ [4] at the outer boundary r_c of the "quasipotential" zone (Fig. 1a),

$$\left(\frac{\partial \bar{\Gamma}}{\partial \eta}\right)\Big|_{\eta=\eta_c} = \frac{\partial}{\partial \eta}\left[\left(\frac{2\eta}{1+\eta^{\varkappa}}\right)^{n}\eta\right]\Big|_{\eta=\eta_c} = 0, \tag{2}$$

$$n = n_c = \frac{1+\eta_c^{\varkappa}}{(\varkappa-1)\eta_c^{\varkappa}-1}. \tag{3}$$

The value of the coefficient κ has been chosen from the best agreement of calculated and test data on the coefficient of twist in the stream core:

$$\varepsilon_c = w_{\varphi m}/w_{\varphi c} = \left(\frac{1+\eta_c^{\varkappa}}{2\eta_c}\right)^{\frac{\eta_c^{\varkappa}+1}{(\varkappa-1)\eta_c^{\varkappa}-1}}. \tag{4}$$

As can be seen from Fig. 1b, the test data are described quite satisfactorily by the equation for a straight line [5]

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